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Fatigue life prediction for simultaneous cyclic loading with blocks of normal stresses and shear stresses

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Abstract. The problem of effects superposition in the case of simultaneous loadings, an important issue in mechanical structures design, have been analised. A method for the multiaxial fatigue life prediction was developed. This method was applied to fatigue life calculation in the cases of cyclical loading with: – one block of normal stress and one bloc of shear stress; – successive blocks of normal stresses, simultaneous with successive blocks of shear stresses. The influence of deterioration, of mean stress and residual stress upon the fatigue life is introduced. The theoretical results: – have been compared with experimental results reported in literature; – may be used for design, as well as for experimental data evaluation. Numerical examples show how the obtained theoretical results may be used in practical cases.

Keywords. Fatigue life, fracture criterion, principle of critical energy, simultaneous cyclic loadings, successive cyclic loadings.

1. Introduction

Fatigue is an important design criteria for mechanical structures subjected to cyclic loading. In this paper new relations are proposed for the fatigue life prediction of mechanical structures, taking into account the mixed mode loading (normal stresses and shear stresses), the mean stresses, residual stress, the deteriorations and the well known mechanical characteristics.
In most cases involving mechanical structure design we refer to the effect caused by a single load. But what happens if several loads are at work?

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Nomenclature:

D_T - total deterioration;
E_k - specific energy of load Y_k;
$E_{k,\mathrm{cr}}$ - critical value of the specific energy E_k;
M_σ, M_τ - material constant;
N - number of cycles up to failure;
P_{cr} - critical participation;
P_k - participation of specific energy E_k;
P_T - total participation of specific energies;
T - period of cycle;
Y_k - the load k;
k; k_1 - material constants;
m - material constant;
n - number of loading cycles;
t - time;
$\alpha = 1/k; \alpha_1 = 1/k_1$; - material constant;
σ - normal stress;
σ_a - normal stress amplitude;
σ_m - mean stress;
$\sigma_{\min}$; $\sigma_{\max}$ - minimum and maximum normal stress;
τ - shear stress;
τ_a - shear stress amplitude;
τ_m - mean shear stress;
$\tau_{\min}$; $\tau_{\max}$ - minimum and maximum shear stress;
τ_{res} - residual shear stress;
σ_u; τ_u - ultimate stresses;
σ_{res} - residual normal stress;
σ_R - normal fatigue limit at the stress ration $R = \sigma_{\min}/\sigma_{\max}$;
τ_R - shear fatigue limit at the stress ration $R = \tau_{\min}/\tau_{\max}$;
σ_{-1}; τ_{-1} - fatigue limits.

In order to solve the problems involving multiple loading one should first distinguish between cases of simultaneous loading (superposition of loadings) and successive loadings (cumulation of loadings).

Superposition of loadings refers to the total effect due to *simultaneous* loading of two or more loadings (Fig. 1, *a*). *Cumulation of loadings* refers to the total effect due to two or more *successive* loadings (Fig. 1, *b*).

The cyclic fatigue loading with *successive* blocks is characterised by stress amplitude $\sigma_{a,i}$ and mean stress $\sigma_{m,i}$, a number of cycles n_i .

The fatigue life calculation under successive loadings with several blocks of normal stresses $\left(\sigma_{a,i};\sigma_{m,i};n_i\right)$ is done by cumulating the deteriorations, for example by using Palmgren-Miner rule [1, 2],

$$\sum_i \left(\frac{n}{N}\right)_i = 1\,, \tag{1}$$

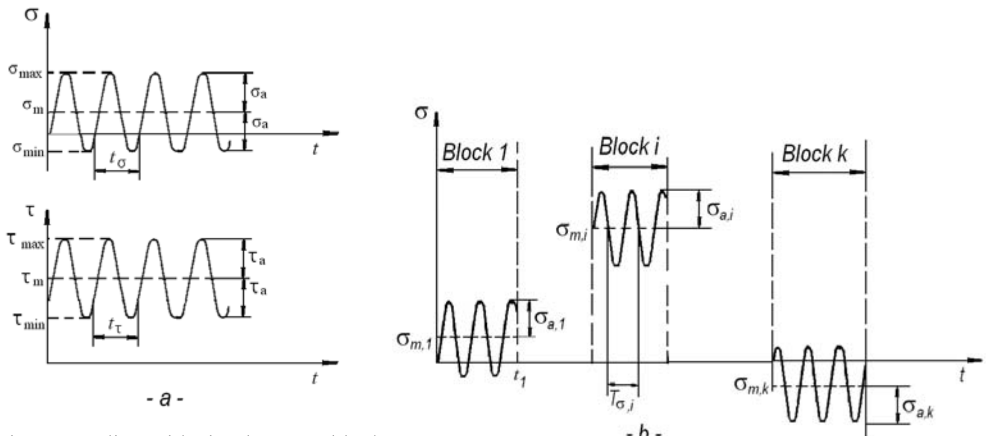

Fig. 1. Loading with simultaneous blocks of cyclic normal stress, σ, and shear stresses, τ, (a) and with successive blocks of cyclic normal stresses (b).

where n_i in the number of loading cycles and N_i is the number of cycles up to failure for the i^{th} stress rang. Palmgren-Miner rule is inaccurate, but its simplicity and the minimal amount of data necessary for implementation makes it a popular method for estimating fatigue life [3]. Several models of multiaxial-block loading, generally modified Palmgren-Miner's law, have been analysed [4]. The state of the art concerning some fatigue life models similar to Palmgren-Miner law (based on cumulative rule) has been discussed in the paper [5]. Meneghetti et al. assume the heat energy as an index of fatigue damage. Minner's rule was applied in terms of energy rather than stress amplitude [6]. Recently there have been established relations for the calculation of the accumulation of the loading action in the case of the *non-linear power law behavior* [7]. Several results reported in literature:

Gladskyi and Fatemi [8] presented the current state of research on the outcome of fatigue loading with axial stresses and, separately, with shear stresses. A few considerations can be gleaned from literature:– in comparison to axial loading, multiaxial fatigue studies are relatively small; – multiaxial fatigue strength is significantly affected by the nominal load ratio [9]; – a significant notch size effect on fatigue life has been observed [10]; – the addition of *static compression* to cyclic torsion generally results in longer lifetime, while *static tension* added to cyclic torsion results in shorts fatigue life in comparison with pure torsion cycling [11]; – the method for fatigue life calculation taking into account the mean stress effect proposed in [12] has as a main limitation the need to know two $S - N$ curves; – the Bauschinger effect may play a significant role on fatigue life of materials and should then be considered in fatigue life prediction models [13]; – tensile pre-straining is beneficial to ratcheting fatigue life, while compressive pre-straining is detrimental [14]; – the fatigue limit value depends on the kind of loading [15]: the ratio between the torsional and the uniaxial fatigue limit (or endurance) is seen to vary in the range 0.5 – 1.0 [16].

Ways of improving the fatigue behavior of welded structures, either during design, or during fabrication, or else by special material characteristics, are addressed in the paper of Wolfgang [17].

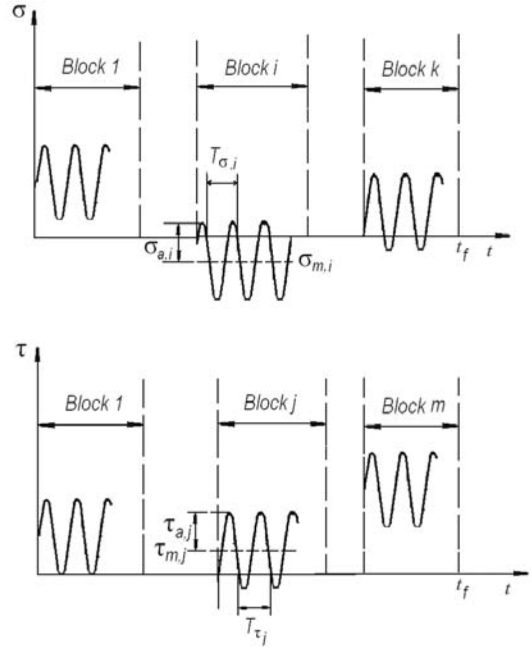

Fig. 2. Simultaneous loading a time t_f, with cyclic normal stresses, σ, with the same period of cycle, $\left(T_{\sigma,i}=T_{\tau,j}\right)$.

Susmel [18] proposes a simple formula suitable for estimating the fatigue strength of welded connections whose weld beads are inclined with respect to the direction along which the fatigue loadings is applied. Fatigue strength is accurately estimated by using the stress components relative to the plane experiencing the maximum shear stress range.

Generally the *multiaxial fatigue* life prediction of engineering materials has been a challenging task for over past decades. To *calculate the lifetime* of a specimen, taken from a material with nonlinear behaviour in the case of *simultaneous loading* with blocks of normal stress and blocks of shear stress, no adequate calculation relation has been found up to now.

This paper, an extension of the paper [5], establishes a general relation for calculating lifetime in the case of superpositions of cyclic fatigue loadings with normal stresses, σ, and shear stresses τ (Fig. 2) for specimens and mechanical structures of materials with nonlinear power-law behavior.

2. Proposal of a new calculation method for fatigue life

2.1. Loading cases

We have signaled out the following loading cases:

– time of cycle, t_σ of normal stress loading is greater than time of cycle, t_τ, of shear stress loading $(t_\sigma > t_\tau)$. In such cases, the superposition of effects with shear and normal stresses, occurs only a number of loading cycles,

$$n_\tau = t_\tau / T_\tau \,, \tag{2}$$

after which the cyclic loading is only with normal stresses, for a number of cycles,

$$n_\sigma(\Delta t) = (t_\sigma - t_\tau)/T_\sigma \,; \tag{3}$$

– time of cycle $t_\sigma < t_\tau$, when the superposition of effects is done for a number of cycles,

$$n_\sigma = t_\sigma / T_\sigma \,, \tag{4}$$

after which the cyclic loading occurs only with shear stresses, for a number of cycles,

$$n_\tau(\Delta t) = (t_\tau - t_\sigma)/T_\tau \,; \tag{5}$$

– the time of cycle of loadings are equal $t_\sigma = t_\tau$, whereupon the number of loading cycles is calculated with relations (2) and (4).

2.2. A new method for fatigue life calculation

Furthermore, one first solves the case $t_\sigma = t_\tau$ when stresses σ and τ are the same frequencies (Fig. 2), by using the *principle of critical energy* presented and used in [5;7; 19-30]. According to this principle total participation of the specific energy involved, P_T, is equal to the sum (cumulation) of specific energy participations corresponding to each load applied, P_k,

$$P_T = \sum_k P_k \,. \tag{6}$$

The individual participation of specific energy is defined as [5],

$$P_k = \frac{E_k}{E_{k,cr}} \cdot \delta_k \,, \tag{7}$$

where E_k is the specific energy (energy of a unit volume, J/m^3 or energy of a unit mass, J/kg) introduced by a load Y_k. $E_{k,cr}$ is the critical value of E_k (it corresponds to fracture);

$\delta_k = 1$, if the load, Y_k, action is in the sense of the fracture process and $\delta_k = -1$, if the load, Y_k, action opposes the fracture process.

In the case of fatigue loading the total participation due to superposition of normal and shear stress action after n loading cycles is given by the relation,

$$P_T(n) = P_T(\sigma; n_\sigma) + P_T(\tau; n_\tau), \tag{8}$$

where,

$$P_T(\sigma; n_\sigma) = \sum_{i=1}^{k} P_i(\sigma; n_\sigma) \text{ and } P_T(\tau; n_\tau) = \sum_{j=1}^{p} P_j(\tau; n_\tau), \tag{9}$$

are the total participation of specific energies due to cumulation of cyclic normal stresses and due to cumulation of cyclic shear stresses, respectively.

One considers the general case of nonlinear behavior according to the following power laws:

$$\sigma = M_\sigma \cdot \varepsilon^k \text{ and } \tau = M_\tau \cdot \gamma^{k_1}, \tag{10}$$

where σ and τ are normal stress and shear stress; ε, γ are strain and shear strain; M_σ, M_τ, k and k_1 are materials constants.

In the case of nonlinear behavior (10), the total participation of the specific energy in the case of loading with only one block of stress is [30]:

- due to normal stress,

$$P_i(\sigma; n) = \left(\frac{\sigma_a}{\sigma_{a,cr}}\right)_i^{\alpha+1} + \left(\frac{\sigma_m}{\sigma_{m,cr}}\right)_i^{\alpha+1} \cdot \delta_{\sigma_{mi}}; \tag{11}$$

- due to shear stress,

$$P_j(\tau; n) = \left(\frac{\tau_a}{\tau_{a,cr}}\right)_j^{\alpha_1+1} + \left(\frac{\tau_m}{\tau_{m,cr}}\right)_j^{\alpha_1+1} \cdot \delta_{\tau_{mj}}, \tag{12}$$

where δ_{σ_m} and δ_{τ_m} are correlated with the mean stress:

$$\delta_{\sigma_{mi}} = \begin{cases} 1, \text{ when } \sigma_{mi} > 0; \\ -1, \text{ when } \sigma_{mi} < 0; \end{cases} \quad \delta_{\tau_{mj}} = \begin{cases} 1, \text{ when } \tau_{mj} > 0; \\ -1, \text{ when } \tau_{mj} < 0. \end{cases}$$

The value of the exponents $\alpha = 1/k$ and $\alpha_1 = 1/k_1$ depend on the material behavior (10) and are influenced by the rate of the external load [31].

The fatigue life calculation, based on he concept of total participation of specific energy, can be done considering the dependence of critical stress value on the number of loading cycles ($\sigma_{cr}(N)$ and $\tau_{cr}(N)$). In this case the participation of the specific energy due to normal stress and shear stress, respectively, after *N* cycles results from relationships (11) and (12) where $\sigma_{a,cr} = \sigma_{-1}(N)$ and $\tau_{a,cr} = \tau_{-1}(N)$. The stresses $\sigma_{-1}(N)$; $\tau_{-1}(N)$ are fatigue strengths after *N* loading cycles with normal and shear stresses, respectively, and $\sigma_{m,cr} = \sigma_u$ and $\tau_{m,cr} = \tau_u$ such as,

$$P_i(\sigma; N) = \left(\frac{\sigma_a}{\sigma_{-1}(N)}\right)_i^{\alpha+1} + \left(\frac{\sigma_m}{\sigma_u}\right)_i^{\alpha+1} \cdot \delta_{\sigma_{mi}}; \tag{13}$$

$$P_j(\tau; N) = \left(\frac{\tau_a}{\tau_{-1}(N)}\right)_j^{\alpha_1+1} + \left(\frac{\tau_m}{\tau_u}\right)_j^{\alpha_1+1} \cdot \delta_{\tau_m}.$$

The simultaneous normal and shear stress loading becomes critical if [32],

$$P_T(n) = P_{cr}(t), \tag{14}$$

where the left side of this Eq. calculates with the Eq. (8), and the right side, the critical participation at the time *t*, is

$$P_{cr}(t) = 1 - D_T(T), \tag{15}$$

with $D_T(t)$ the total deterioration due to cracks, preloading, corrosion, creep, aging etc. [32]. En consequence one can write,

$$\sum_{i=1}^{k} P_i(\sigma; n_\sigma) + \sum_{j=1}^{p} P_j(\tau; n_\tau) = 1 - D_T(t), \tag{16}$$

which is *a fracture criterion*.

The fatigue curve is described by Basquin's law [33],

$$\sigma_a^m \cdot N = A, \tag{17}$$

where *A* and *m* are constants corresponding to each domain of the fatigue curve (Fig. 3).

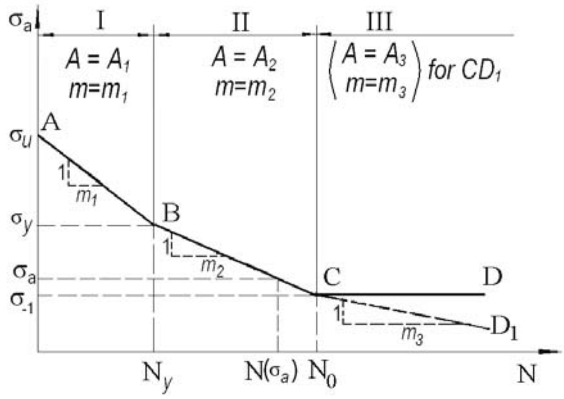

Fig. 3. Fatigue (Wöhler) curve: I; II; III – the three domains of the fatigue curve.

For the normal stress fatigue curve, for example slope m_2 (domain II) is usually assumed to be a constant for steels, for both welded joints and base material $(m_2 = 3)$, as well as for aluminium alloy welded joints $(m_2$ changes from class to class between 3.4 and 4.3).

3. Fatigue life criterion in the case of loading with cyclical blocks of normal stresses and shear stresses

a. From the relation (8), (9), (13) - (16) it has been obtained [5] the fatigue life due to several block of normal stresses $(\sigma_{a,1};\sigma_{m,1}),(\sigma_{a,2};\sigma_{m,2})\ldots(\sigma_{a,i};\sigma_{m,i})\ldots$,

$$\sum_i \left[\frac{n_\sigma}{N(\sigma_a)}\right]_i^{\frac{\alpha+1}{m}} = C_\sigma, \tag{18}$$

where $n_{\sigma,i}$ is the effective number of cycles with the normal stress amplitude $\sigma_{a,i}$, while $(N(\sigma_a))_i$ is the fatigue life for the i^{th} normal stress amplitude $(\sigma_{a,i})$ and

$$C_\sigma = 1 - \left(\frac{\sigma_m}{\sigma_u}\right)_f^{\alpha+1} \cdot \delta_{\sigma_m} - D_T(t) \tag{19}$$

where σ_m is the mean normal stress, $(\sigma_m/\sigma_u)_f$ is the value of this ratio in the final (last) block of normal stress loading. If one considers the local residual normal stress, σ_{res}, one add up the term $\left[-\left(\frac{\sigma_{res}}{\sigma_u}\right)^2\cdot\delta_{\sigma_{res}}\right]$ to the Eq. (19) of C_σ.

The individual fatigue life, $N(\sigma_a)$ in Eq. (18), for each normal stress amplitude σ_a (Fig. 3), can be calculated using the following equations [5]:

– for diagram $\sigma_a - N$,

$$N(\sigma_a)=\begin{cases} N_y\cdot\left(\dfrac{\sigma_y}{\sigma_a}\right)_{\mathrm{I}}^{m_1} & \text{- for domain I } (\sigma_y \le \sigma_a < \sigma_u);\\ N_0\cdot\left(\dfrac{\sigma_{-1}}{\sigma_a}\right)_{\mathrm{II}}^{m_2} & \text{- for domain II } (\sigma_{-1} < \sigma_a < \sigma_y);\\ N_0\cdot\left(\dfrac{\sigma_{-1}}{\sigma_a}\right)_{\mathrm{III}}^{m_3} & \text{- for curve } CD_1 \text{ in domain III } (\sigma_a \le \sigma_{-1}); \end{cases} \tag{20}$$

– for diagram $\sigma_{\max} - N$,

$$N(\sigma_a)=\begin{cases} N_y\cdot\left(\dfrac{\sigma_y}{\sigma_{\max}}\right)_{I}^{m_1} & \text{- for domain } I \ (\sigma_y \le \sigma_{\max} < \sigma_u);\\ N_0\cdot\left(\dfrac{\sigma_R}{\sigma_{\max}}\right)_{\mathrm{II}}^{m_2} & \text{- for domain II } (\sigma_R < \sigma_{\max} < \sigma_y);\\ N_0\cdot\left(\dfrac{\sigma_R}{\sigma_{\max}}\right)_{III}^{m_3} & \text{- for domain III } (\sigma_{\max} \le \sigma_R), \end{cases} \tag{21}$$

where σ_R is the fatigue limit at the stress ratio $R=\sigma_{\min}/\sigma_{\max}$. In general [5],

$$\sigma_R=\left(\sigma_{-1}^{\alpha+1}+\sigma_m^{\alpha+1}\right)^{1/(\alpha+1)}. \tag{22}$$

b. In the case of several blocks of shear stresses $(\tau_{a,1};\tau_{m,1}),(\tau_{a,2};\tau_{m,2})\ldots(\tau_{a,j};\tau_{m,j})\ldots$ for a nonlinear behavior given by the second law (10), by a similar procedure as in the case of normal stresses one obtains,

$$\sum_j\left(\frac{n_\tau}{N(\tau_a)}\right)_j^{\frac{\alpha_1+1}{m}}=C_\tau, \tag{23}$$

where $n_{\tau,j}$ is the effective number of cycles with the shear stress amplitude $\tau_{a,j}$, while $(N(\tau_a))_j$ is the fatigue life for the j^{th} shear stress amplitude $(\tau_{a,j})$ and

$$C_\tau=1-\left(\frac{\tau_m}{\tau_u}\right)_f\cdot\delta_{\tau_m}-D_T(t). \tag{24}$$

where τ_m is the mean shear stress; $(\tau_m/\tau_u)_f$ in the value of this ratio in the final (last) block of shear stress loading. If one considers the local residual shear stress, τ_{res}, one add up the term $\left[-\left(\frac{\tau_{res}}{\tau_u}\right)^2\cdot\delta_{\tau_{res}}\right]$ to the Eq. (24) of C_τ.

By a similar procedure as for the normal stress amplitude, one may write:

– for $\tau_a - N$ *diagram*,

$$N(\tau_a)=\begin{cases} N_y\cdot\left(\dfrac{\tau_y}{\tau_a}\right)_{\mathrm{I}}^{m_1} & \text{- for domain } I \ \left(\tau_y \le \tau_a < \tau_u\right); \\ N_0\cdot\left(\dfrac{\tau_{-1}}{\tau_a}\right)_{II}^{m_2} & \text{- for domain II}\left(\tau_{-1} < \tau_a < \tau_y\right); \\ N_0\cdot\left(\dfrac{\tau_{-1}}{\tau_a}\right)_{\mathrm{III}}^{m_3} & \text{- for curve CD}_1 \text{ domain } III \ \left(\tau_a \le \tau_{-1}\right), \end{cases} \tag{25}$$

where N_y and N_0 as well as m_1, m_2, m_3 correspond to the fatigue curve $\tau_a - N$, similar to curve $\sigma_a - N$;

– for $\tau_{\max} - N$ *diagram*,

$$N(\tau_a)=\begin{cases} N_y\cdot\left(\dfrac{\tau_y}{\tau_{\max}}\right)_{\mathrm{I}}^{m_1} & \text{- for domain I } \left(\tau_y \le \tau_{\max} < \tau_u\right); \\ N_0\cdot\left(\dfrac{\tau_R}{\tau_{\max}}\right)_{\mathrm{II}}^{m_2} & \text{- for domain II}\left(\tau_R < \tau_{\max} < \tau_y\right); \\ N_0\cdot\left(\dfrac{\tau_R}{\tau_{\max}}\right)_{\mathrm{III}}^{m_3} & \text{- for domain III, curve CD}_1 \ \left(\tau_{\max} \le \tau_R\right). \end{cases} \tag{26}$$

In general,

$$\tau_R = \left(\tau_{-1}^{\alpha_1+1} + \tau_m^{\alpha_1+1}\right)^{\frac{1}{\alpha_1+1}}. \tag{27}$$

c. In the case *of cyclic* loading by both (superposition), *blocks of normal and blocks of shear stresses* (Fig. 2), with $t_\sigma = t_\tau = t_f$, taking into account the principle of critical energy, one may write the following general relation:

$$\sum_i\left(\frac{n_\sigma}{N(\sigma_a)}\right)_i^{\frac{\alpha+1}{m}} + \sum_j\left(\frac{n_\tau}{N(\tau_a)}\right)_j^{\frac{\alpha_1+1}{m}} = C, \tag{28}$$

where for the sample without residual stresses $(\sigma_{res} = 0)$,

$$C = \begin{cases} \min(C_\sigma; C_\tau) \text{ - if the loading by } \sigma \text{ and } \tau \text{ is successive;} \\ 1-\left(\dfrac{\sigma_m}{\sigma_u}\right)_f^{\alpha+1}\cdot\delta_{\sigma_m}-\left(\dfrac{\tau_m}{\tau_u}\right)_f^{\alpha_1+1}\cdot\delta_{\tau_m}-D_T(t) \text{ - if the loading by } \sigma \text{ and } \tau \text{ is} \\ \text{simultaneous (Fig. 2).} \end{cases} \tag{29}$$

Due to $(\sigma_m/\sigma_u)_f$ and $(\tau_m/\tau_u)_f$ rations the relation (18), (23) and (28) describes the influence of load sequence, load level and load interaction effects.
On the other side, these theoretical relationships, based on the physical behavior of the sample, do not contain empirical constants, which, generally, results by fitting the proposed Eq. with the experimental results.

4. Experimental confirmations

Tubular type 304 stainless steel sample have been obtained as follows [34]: a bar was cut to the specimen length, then the inner hole was gun – drilled and honed through the center of the bar, the external surface was machined and finally, the gage section was ground and polished with aluminia powder. These mechanical processes introduces in the sample material, normal residual stresses $\sigma_{res} < 0$ (compression stresses). En consequence *C* from Eq. (29) becomes C_{res}, respectively

$$C_{res} = C - \left(\frac{\sigma_{res}}{\sigma_u}\right)^2. \tag{30}$$

In the case of cyclic loading path, tension - compression followed by torsion, Eq. (28) becomes

$$\left(\frac{n_\sigma}{N(\sigma_a)}\right)^{\frac{\alpha+1}{m}} + \left(\frac{n_\tau}{N(\tau_a)}\right)^{\frac{\alpha_1+1}{m}} = 1 - \left(\frac{\sigma_{res}}{\sigma_u}\right)^2, \tag{31}$$

because the sample was undeteriorated $(D_T = 0)$, and the cyclic loadings have been fully reversed ($\sigma_m = 0$ and $\tau_m = 0$). The sum in the left part of Eq. (31) is less than one. The experimental results show the same $C_{res} < 1$ [35].
Nowadays the fatigue life may be calculated by Palmgren – Miner's cumulative fatigue damage rules (1). This eq. always predicted a longer fatigue life than the observed one [2].
With $(\alpha+1)/m$ and $C_\sigma = 1$ the eq. (18) becomes Palmegren – Miner's rule (1). One can observe [5]:
– in the case of a sample without residual stresses $(\sigma_{res} = 0)$ and undeteriorated $(D_T = 0)$:

$C_\sigma = 1$, in only a particular case, namely for alternating symmetrical loading $(R = -1)$ when $\sigma_m = 0$;
$C_\sigma < 1$, if $\sigma_m > 0$ and $C_\sigma > 1$, if $\sigma_m < 0$ $(R \neq -1)$;
– in the case of a sample without residual stresses $(\sigma_{res} = 0)$ but cracked and pre-loaded sample (when $D_T \neq 0$), $C_\sigma < 1$, for alternating symmetrical loading $(R = -1)$; $C_\sigma < 1$ or $C_\sigma > 1$ in the general case with $R \neq -1$.
Experimentally it has observed that C takes values that can be either less or higher the unity. Miner [2] has shown that fracture occurred at $C_\sigma = 0.61 - 1.49$. A dispersion of C_σ values between 0.2 and 3.0 was obtained. Numerous tests have shown that values of C_σ at failure may deviate considerably from unity, taking values between 0.1 and 10 [35].
Some experimentally determined values for the exponent in eq. (18) are equal [1;2] to 0.6…1.0. For some steels have been found [5] the exponent $(\alpha+1)/m = 0.2...2.4$. En consequence, generally, the exponent $(\alpha+1)/m \neq 1$ and $C_\sigma \neq 1$, as it results from eqs. (18) and (19), as well from literature [36].
A static stress as well as prestresing stress adds to the mean stress and consequently modified the value of C_σ or C_τ. Through these variables the fatigue life modifies as it results from eqs. (19), (24) and (29) and as it was reported in literature [11; 15].

5. The influence of mean stress on the fatigue life

The value of the right term in the relations (18), (23) and (28) influences the life time. In the case of undeteriorated sample $(D_T = 0)$ loaded with normal stresses, from Eq. (19) results,

$$C_\sigma = 1 - \left(\frac{\sigma_m}{\sigma_u}\right)_f^{\alpha+1} \cdot \delta_{\sigma_m}, \qquad (32)$$

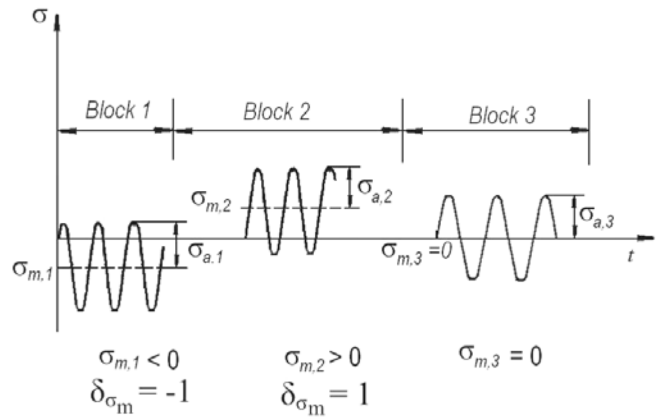

Fig. 4. Three different blocks of cyclic normal stresses.

If the mean stress is a compression stress $\left(\sigma_{m_1} < 0\right)$, $\delta_{\sigma_m} = -1$, such as $C_\sigma > 1$. But, if the mean stress is a tension stress $\left(\sigma_{m_2} > 0\right)$, than $\delta_{\sigma_m} = 1$, such as $C_\sigma < 1$. For alternating symetrical loading $\left(\sigma_{m_3} = 0\right)$ results $C_\sigma = 1$. This may explain why the succession of loads influences the result. For example (Fig. 4):
– the loading such as the final block of stresses is the block 3 $\left(\sigma_{m,3} = 0\right)$, gives $C_\sigma = 1$;
– the loading with a final block like the block 2 $\left(\sigma_{m,2} > 0\right)$, gives $C_\sigma < 1$;
– the loading with a final block like the block 1 $\left(\sigma_{m,1} < 0\right)$, gives $C_\sigma > 1$.
In the case of undeteriorated sample $\left(D_T = 0\right)$, loaded with shear stresses, from Eq. (24) results,

$$C_\tau = 1 - \left(\frac{\tau_m}{\tau_u}\right)_f \cdot \delta_{\tau_m}, \tag{33}$$

The discussion about the influence of the mean shear stress, τ_m, upon C_τ is similar to above one for σ_m. It must only replace σ_m by τ_m.

6. Numerical examples

1. A shaft is made of steel featuring the following mechanical properties:
ultimate stresses, $\sigma_u = 640\,\text{MPa}$ and $\tau_u = 460\,\text{MPa}$; yield stresses, $\sigma_y = 386\,\text{MPa}$ and $\tau_y = 240\,\text{MPa}$; fatigue limits $\sigma_{-1} = 290\,\text{MPa}$ and $\tau_{-1} = 195\,\text{MPa}$; $N_y = 10^4$ cycles; $N_0 = 2\times10^6$ cycles; $k = 0.25$; $k_1 = 0.25$; $m_1 = 2.5$ and $m_2 = 3.5$.
The shaft is undeteriorated, $D_T(t) = 0$.
We consider two different cases of fatigue loading. Each of them is analysed to establish if the fatigue load is or is not dangerous.
a. The shaft is loaded by simultaneous two blocks of cyclic normal stresses (Table 1) and two blocks of shear stresses (Table 2).

Table 1

	The block of loading, i		1	2
1.	Normal stress, MPa	σ_{max}	450	360
		σ_{min}	0	-360
2.	Number of cyclic loadings in each block, $n_{\sigma,i}$		2×10^3	10^5
3.	Domain of loading (Fig. 3)		I	II
4.	Stress amplitude, $\sigma_{a,i}$, MPa		225	360
5.	Mean stress, $\sigma_{m,i}$, MPa		225	0
6.	Number of cycles to fatigue failure, $N_i(\sigma_a)$		0.681×10^4	0.938×10^6

For *the first block* of normal stresses the stress amplitude and the mean stress are, as follows:

$\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min}) = 0.5(450 - 0) = 225$ MPa ;

$\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min}) = 0.5(450 + 0) = 225$ MPa .

Because $\sigma_{\max} > \sigma_y$ the loading corresponds to the domain I (Fig. 3). Out of relationship (18)

$$P(n_\sigma) = \left(\frac{n_\sigma}{N\,(\sigma_a)}\right)_{\mathrm{I}}^{\frac{\alpha+1}{m}} = \left(\frac{2\times 10^3}{0.681\times 10^4}\right)_{\mathrm{I}}^{\frac{5}{2.5}} = 0.08625,$$

where $\alpha = 1/k = 1/0.25 = 4$ and out of relationship (21), for the first (1) block of loading (Table 1) results,

$$N(\sigma_a) = N_y \cdot \left(\frac{\sigma_y}{\sigma_{\max}}\right)_{\mathrm{I}}^{m_1} = 10^4 \times \left(\frac{386}{450}\right)^{2.5} = 0.681\times 10^4 \text{ cycles.}$$

For the *second block* (2) of loading (Table 1), out of relationship (20) results:

$$N(\sigma_a) = N_0 \cdot \left(\frac{\sigma_{-1}}{\sigma_a}\right)_{\mathrm{II}}^{m_2} = 2\times 10^6 \times \left(\frac{290}{360}\right)^{3.5} = 0.938\times 10^6 \text{ cycles.}$$

These values of $N(\sigma_a)$ are show in Table 1.

For the blocks of cyclically *shear stress* (Table 2) loading the number of cycles to fatigue failure are:

$$N(\tau_a) = N_0 \cdot \left(\frac{\tau_{-1}}{\tau_a}\right)_{\mathrm{II}}^{m_2} = 2\times 10^6 \times \left(\frac{195}{300}\right)^{3.5} = 4.428\times 10^5 \text{ cycles} - \text{in the domain II}$$

(Eq.25) for the first block (1) of loading (Table 2);

Table 2

	The block of loading, *j*		1	2
1.	Shear stress, MPa	$\tau_{\max}$	300	250
		$\tau_{\min}$	-300	100
2.	Number of cyclic loadings in each block, $n_{\tau,j}$		2×10^5	10^5
3.	Domain of loading		II	II
4.	Shear stress amplitude, $\tau_{a,j}$, MPa		300	75
5.	Mean shear stress, $\tau_{m,j}$, MPa		0	175
6.	Number of cycles to failure, $N_j(\tau_a)$		4.428×10^5	7.905×10^5

$$N(\tau_a) = N_0 \cdot \left(\frac{\tau_R}{\tau_{\max}}\right)_{II}^{m_2} = 2\times 10^6 \times \left(\frac{233.76}{250}\right)^{3.5} = 7.905\times 10^5 \text{ cycles} - \text{in the domain}$$

II (Eq. 26) for the second (2) block of loading (Table 2), where according to eq. (27), with $\alpha_1 = 1/k_1 = 1/0.25 = 4$,

$\tau_R = \left(\tau_{-1}^{\alpha_1+1} + \tau_m^{\alpha_1+1}\right)^{\frac{1}{\alpha_1+1}} = \left(195^5 + 175^5 \cdot 1\right)^{\frac{1}{5}} = 213.74\ \text{MPa}.$

These values of $N(\tau_a)$ are shown in Table 2.

Out of the left part of the Eq. (28),

$$P_T(n) = \sum_{i=1}^{2}\left(\frac{n_\sigma}{N(\sigma_a)}\right)_i^{\frac{\alpha+1}{m}} + \sum_{j=1}^{2}\left(\frac{n_\tau}{N(\tau_a)}\right)_j^{\frac{\alpha_1+1}{m}} = \left[\left(\frac{2\times10^3}{0.681\times10^4}\right)^{\frac{5}{2.5}} + \left(\frac{10^5}{0.938\times10^6}\right)^{\frac{5}{3.5}}\right]_\sigma +$$

$$+\left[\left(\frac{2\times10^5}{4.428\times10^5}\right)^{\frac{5}{3.5}} + \left(\frac{10^5}{7.905\times10^5}\right)^{\frac{5}{3.5}}\right]_\tau = 0.12709 + 0.37343 = 0.50052.$$

Because the σ and τ cyclic loadings are simultaneous, out of relationship (29),

$$C = 1 - \left(\frac{\sigma_m}{\sigma_u}\right)_f^{\alpha+1} \cdot \delta_{\sigma_m} - \left(\frac{\tau_m}{\tau_u}\right)_f^{\alpha+1} \cdot \delta_{\tau_m} = 1 - \left(\frac{0}{640}\right)^5 - \left(\frac{175}{460}\right)^5 = 0.992.$$

Because $P_T(n) < C$ - the fatigue loading is not dangerous.

2. The same shaft as in the first example is loaded by cyclic normal and shear stresses. The shaft has a crack $a(n) = 3$ mm whose critical value due to normal stresses is $a_{cr} = 10\ \text{mm}$. The number of both cyclic loadings $n_\sigma = n_\tau > N_0 = 2\times10^6$. The shaft steel is characterised by fatigue limits σ_{-1} and τ_{-1} (curve CD in Fig. 3). The shaft is cyclically loaded simultaneous by normal and shear stresses:

$\sigma_{\max} = 250\ \text{MPa}$; $\sigma_{\min} = -250\ \text{MPa}$; $\tau_{\max} = 200\ \text{MPa}$; $\tau_{\min} = 0$. As a result,

- the normal stress amplitude $\sigma_a = 0.5(\sigma_{\max} - \sigma_{\min}) = 250\ \text{MPa}$;
- the normal mean stress $\sigma_m = 0.5(\sigma_{\max} + \sigma_{\min}) = 0$;
- the shear stress amplitude $\tau_a = 0.5(\tau_{\max} - \tau_{\min}) = 100\ \text{MPa}$;
- the shear mean stress $\tau_m = 0.5(\tau_{\max} + \tau_{\min}) = 100\ \text{MPa}$.

Out of relationships (8) and (13) - (16), one obtains,

$$\left(\frac{\sigma_a}{\sigma_{-1}}\right)^{\alpha+1} + \left(\frac{\tau_m}{\tau_u}\right)^{\alpha_1+1} = B, \tag{34}$$

where

$$B = 1 - \left(\frac{\sigma_m}{\sigma_u}\right)^{\alpha+1} \cdot \delta_{\sigma_m} - \left(\frac{\tau_m}{\tau_u}\right)^{\alpha_1+1} \cdot \delta_{\sigma_m} - D_T(t), \tag{35}$$

The left part of relation (34) becomes,

$$P_T(n) = \left(\frac{\sigma_a}{\sigma_{-1}}\right)^{\alpha+1} + \left(\frac{\tau_a}{\tau_{-1}}\right)^{\alpha_1+1} = \left(\frac{250}{290}\right)^5 + \left(\frac{100}{195}\right)^5 = 0.51157,$$

while,

$$B = 1 - \left(\frac{0}{640}\right)^5 - \left(\frac{100}{460}\right)^5 - \left(\frac{3}{10}\right)^{\frac{5}{2}} = 0.9725\,,$$

where the deterioration due to the crack $a = 3\,\mathrm{mm}$ is [32],

$$D_T(t) = \left(\frac{a}{a_{cr}}\right)^{\frac{\alpha+1}{2}} = \left(\frac{3}{10}\right)^{\frac{5}{2}}.$$

Because $P_T(n) < B$ this loading by cyclic normal stress and cyclic shear stress is not dangerous.

7. Conclusion

In the paper the problem of fatigue life was discussed. A new model of fatigue life calculation was proposed based on the principle of critical energy. It takes into account the nonlinear behaviour of the loaded material, the mean stress and its sign, the deterioration and the residual stress.
Using non-dimensional concepts (introduced by the principle of critical energy) and defining them in the case of fatigue loading of nonlinear materials, a unitary theory was established for the fatigue life of sample under cyclic loading produced by several blocks of normal stresses (18), by several blocks of shear stresses (23) and by superposition of several blocks of normal stresses and by several blocks of shear stresses (28).
Numerical examples show how to use the proposed fatigue life rule.

References

[1] Palmgren A., *Die lebensdauer von Kugellagern*, Zeitschr, 68 (1924) 339-410.
[2] Miner M.A., *Cumulative damage in fatigue*, J. Appl. Mech., 67 (1945) A159 – A164.
[3] Huffman P.J., Beckman S.P., *A non linear damage accumulation fatigue model for predicting strain life at variable amplitude loading based on constant amplitude fatigue data*, Int. J. Fatigue, 48 (2013) 165-169.
[4] Xia T-X, Yao W-X. (2013). *Comparative research on the accumulative damage rules under multiaxial block loading spectrum for 2024-T4 aluminium alloy*, Int. J. Fatigue, 48, 257-265.
[5] Jinescu V.V., *Critical Energy Approach for the Fatigue life Calculation under Blocks with different normal Stress Amplitudes*, Int. J. Mechanical Sci., 67 (2013) 78 – 88.
[6] Meneghetti G., Ricotta M., Atzori B., *Experimental evalution of fatigue damage in two-stage loading tests based on the energy dissipation*, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 9 (2015) 1280-1291.
[7] Jinescu V.V., *The Principle of Critical Energy, Consequences and Applications*, Proceedings of the Romanian Academy, Series A. vol. 14, 2 (2013) 152-160.
[8] Gladskyi M., Fatemi A., *Notched fatigue behaviour including load sequence effects under axial and torsional loadings*, Int. J. Fatigue, 55 (2013) 43-53.

[9] Atzori B., Berto F., Lazzarin P., Quaresimin M., *Multiaxial fatigue behaviour of a severely notched carbon steel*, Int. J. Fatigue, 28 (2006) 486-493.
[10] Gao Z., Qui B., Wang X, Jiang Y., *An investigation of fatigue of a notched member*, Int. J. Fatigue, 32 (2010) 1960-1969.
[11] Thomson K.D., Sheppard S.D., *Fatigue crack growth in notched and plain shafts subjected to torsion and axial loading,* Eng. Fract.Mech., 43 (1992) 55-71.
[12] Niesony A., Bőhm M., *Mean stress effect correction using constant stress ratio S-N curves*, Int. J. Fatigue, 52 (2013) 49-56.
[13] Buciumeanu A., Palaghian L., Miranda A.S., Silva F.S., *Fatigue life predictions including the Buschinger effect*, Int. J. Fatigue, 33 (2011) 145-152.
[14] Wang Y., Yu D., Chen G., Chen X*., Effects of pre-stain on uniaxial ratcheting and fatigue failure of Z2 CN18.10 austenitic stainless steel*, Int. J. Fatigue, 52 (2013) 106-113.
[15] Susmel L., Taylor D., *The theory of critical distances to estimate finite lifetime of notched components subjected to constant and variable amplitude torsional loading*, Eng. Fracture Mech., 2013, 64-79.
[16] Susmel L., Lazzarin P., *A bi-parametric modified Wöhler curve for high cycle multiaxial fatigue assessment*, Fat. Fracture Engng Mat Structures, 25 (2002) 63-78.
[17] Wolfgang F., *Recent developments and future challenges in fatigue strength assessment of welded joints*, Proceedings of the Institution of Mechanical Engineers, Part C., Journal of Mechanical Engineering Science, 229 (2015) 1224-1239.
[18] Susmel L., *Nominal stresses and Modified Wőhler Curve Method to perform the fatigue assessment of uniaxially loaded inclined welds*, Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 228 (2014) 2871-2880.
[19] Jinescu V.V., *Principiul energiei critice*, Revista de Chimie, 35, nr. 9 (1984) 858-861.
[20] Jinescu V.V., Iordăchescu Vali-Ifigenia *Calculation of deterioration due to cracks in tubular specimens*, U.P.B. Sci.Bull., Series D, vol. 76 (2014) 149 – 160.
[21] Jinescu V.V., Iordăchescu Vali-Ifigenia, Teodorescu Nicoleta, *Relation for Calculation of Critical Stresses in Pressure Equipment with Cracks*, Revista de Chimie, 64, nr. 8 (2013) 858-863.
[22] Jinescu V.V., *Effects Superposition by Buckling, Fatigue and Creep*, International Journal of Pressure Vessels and Piping, 53, nr. 3 (1993) 377-391.
[23] Jinescu V.V., *Stability Determination of Structure under Groups of Loads by Using the Principle of Critical Energy*, International Journal of Pressure Vessels and Piping, 48 (1991) 343-375.
[24] Jinescu V.V., *Principiul energiei critice, VIII. Solicitări compuse în condiţii de fluaj*, Revista de Chimie, 41, nr. 10 (1990) 769-771.
[25] Jinescu V.V., *Energonica*, Editura Semne, Bucureşti, 1997.
[26] Jinescu V.V., *Principiul energiei critice III. Aplicaţii în cazul solicitărilor variabile izoterme*, Revista de Chimie, 40, nr. 1 (1989) 67-72.
[27] Jinescu V.V., *Tratat de Termomecanică,* vol. 1, Editura AGIR, Bucureşti, 2012.
[28] Jinescu V.V., *Effects Superposition by Buckling, Fatigue and Creep*, International Journal of Pressure Vessels and Piping, 53, nr. 3 (1993) 377-391.
[29] Jinescu V.V., *Contribuţii la calculul deteriorării materialelor şi echipamentelor de proces*, II, Revista de Chimie, 59, nr. 4 (2008) 453-461.
[30] Jinescu V.V., *Principiul energiei critice şi aplicaţiile sale*, Editura Academiei Române, Bucureşti, 2005.
[31] Faupel J.H., *Engineering Design*, John Wiley, New York, 1964.
[32] Jinescu V.V., *Cumulation of Effects in calculation the Deterioration of Fatigue loaded Structures*, Int J Damage Mech, 21 (2012) 671-695.
[33] Basquin O.H., *The exponent low of endurance tests*, Proc. Am. Soc. Test Mater. 10 (1910) 625-630.
[34] Chen X., Jin D., Kim K.S., *Fatigue life prediction of type 304 stainless steel under sequential biaxial loading*, Int. J. Fatigue, 28 (2006) 289-299.
[35] Buch A., *Fatigue strength calculation*. Switzerland, Trans. Tech. Publications, 1988.
[36] Jinescu V.V., *Application in Mechanical Engineering of Principle of Critical Energy*, Lambert Academic Publishing, Saarbrücken, Germany, 2015.